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THRESHOLD CONCEPTS IN SOLVING QUADRATIC EQUATIONS USING FACTORIZATION METHOD

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Abstract: **A descriptive survey research design and a mixed method research paradigm was employed in this study. Quantitative data collected was analyzed using both descriptive and inferential statistics based on the objectives of the study. The study found the following threshold concepts: Also imposing linear structure, not dividing the equation by coefficient of x^2, not adding half the coefficient of x and being unable to convert the solution to squared form. Considering these concepts, teachers should emphasize these threshold concept during teaching.**

Keywords: **Comprehension, Performance, Threshold Concepts and Quadratic Equations.**

1. INTRODUCTION

Quadratic equations are one of the most conceptually challenging aspects of the high school curriculum. This is because many secondary learners lack comprehension on factorization especially with basic multiplication table fact retrieval (Kotsopoulos, 2007). Factorization concept is a process of finding products within the multiplication table; this directly influences students' ability to engage effectively in factorization of quadratics. Furthermore, most secondary school learners were found to be confused about the concept of a variable and the meaning of a solution to a quadratic equation. For example, even if most students were able to obtain the correct solutions, x = 3 and x = 5 learners thought that the two xs in the equation (x – 3) (x – 5) = 0 stood for different variables. This showed that the students lack relational understanding and relied only on rote learning (Law & Shahrill, 2013; Pungut & Shahrill, 2014; Sarwadi & Shahrill, 2014; Vaiyavutjamai, 2004; Vaiyavutjamai, Ellerton & Clements, 2005; Vaiyavutjamai & Clements, 2006 as cited in Yahya & Shahrill, 2015).

In solving quadratic equations and functions, learners must choose and employ a correct technique in order to get a correct solution. The correct and incorrect technique commonly employed by learners to solve problems includes changing the subject of a given formula, factorizing quadratic expressions and solving quadratic equations using the formula Yahya and Shahrill, (2015). Some of the concepts include the failure to manipulate operations correctly in changing the subject of a given formula, the incorrect selection of multiplication factors in the factorization of quadratic expressions, and the inability to recall correct quadratic formula in solving quadratic equations. There were other psychological factors noted as well, such as carelessness and participants' lack of confidence in answering the questions. Learner's performance was compared with gender and school type in order to describe their influence.

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2. FACTORIZATION IN SOLVING QUADRATIC EQUATIONS

Threshold concepts in solving quadratic equations can be identified when learners used the methods of factorization, completing the square, quadratic formula or the graphical method.

2.1 Factorization

There seems to be agreement in the field that when solving equations, students tend to use procedures without understanding and that students have difficulties with aspects of solving quadratic equations such as factoring, applying the zero-product property, and solving equations that are not in general form (Didis & Erbas, 2015). Learners' approach to solving quadratic functions and equations prefer factoring as a solution method when the quadratic is obviously factorable, and yet, factoring can be tricky for learners, particularly when the leading coefficient does not equals to 1 (Nielsen, 2015).

Zakaria and Maat (2010), while conducting a case study that used a survey method on 30 Form Three students grouped in three different category of achievement: low, medium and high. The school has classified learners' achievement based on placement test. Three (3) male and four (4) female learners belong to high category, the medium category has 5 males and 7 females and the low category has 6 males and 5 females. The results of the study was that, most of the types of errors made by learners in using factorization to determine the root of a quadratic equation were transformation errors followed by process skill errors. Factoring can be problematic for students as claimed by (Nielsen, 2015). Some learners have difficulties with their multiplication facts, which make it difficult for them to quickly find factors for expressions in the form Kotsopoulos, (2007). These difficulties increase when the parameter a does not equal one (for example in expressions such as $6x^2+3x+2$ and become even more challenging when a and/or c have multiple factors, leading to many possible factor pairs in expressions such as $20x^2+63x+36$. It is worth noting that the research literature on factoring quadratics attends to factoring when a, b, and c are integers resulting in expressions that can be factored into binomials with integer coefficients.

Mamba, (2012) conducted a research study which sought to get a deep understanding of why learners continue to perform poorly, and what the factors are which contribute to poor performance. This research entailed a detailed error analysis of four items of the 2008 Mathematics paper 1 senior certificate examination scripts, to see the trends and patterns of written responses with regards to the types of errors made by learners. The study was aimed at investigating South African Grade 12 learners' errors exhibited when solving quadratic equations, quadratic inequalities and simultaneous equations. The four items analyzed in the study comprised of questions from three important areas of algebra namely: quadratic equations, quadratic inequalities and simultaneous equations. The scripts were analyzed for carelessness, conceptual and procedural errors.

The learner misconceptions were discovered in learners' work; these comprised the notions of equality and inequality, the construct of the variable, order of operations, factorization, and solution of equations instead of inequalities. From this, the researcher noted that learners' learning difficulties are usually presented in the form of errors they show. Not all the errors that learners had are the same; some errors in procedures can simply be due to learners' carelessness or overloading working memory. The results obtained indicated a number of error categories under each conceptual area, namely, quadratic equations and inequalities and simultaneous equations. Under the conceptual areas indicated above, the main reason for misconceptions seemed to be the lack of understanding of the basic concepts including numbers and numerical operations; functions; the order of operations; equality; algebraic symbolism; algebraic equations, expressions and inequalities; and difference between equations, expressions and inequalities.

Didis and Erbas, (2015) in their research on the performance and difficulties of students in formulating and solving quadratic equations with one unknown utilized mixed method research design. It draws on both qualitative and quantitative data to describe and analyze learners' performances and difficulties with quadratic equations in symbolic and word-problem contexts. The participants were 217 tenth grade learners from three public Anatolian high schools in Turkey; one is located in Ankara ($n_1 = 84$) and the other two are in Çorum ($n_2 = 78$ and $n_3 = 55$). The participants' ages ranged from 14 to 16 (mean = 15.95 and $sd = 0.46$). The quantitative analysis of the data revealed that only about ten percent of the learners ($N = 217$) solved all of the symbolic equation questions correctly. The data shows that the percentage of correct solutions ranged from 25.8% to 80.6%.

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Makonye and Shingirayi, (2014) on their research on obstacles faced by the learners in the learning of quadratic inequalities, discovered that learner errors on quadratic inequalities tasks lie in their lack of competency on basic algebraic processes. The research also found that in most cases learners made different kinds of errors in response to a single problem. This indicates that the process of error making is not static. More than 80% of the learners were not in a position to solve inequalities due to failure in algebraic processes, such as factorization, transposing inequalities so that one side of the inequality became zero. In other cases, learners failed to determine the critical values of the quadratic equations inherent in the inequality. Where learners were able to transpose, they had challenges in assigning the correct signs. In some cases learners relegated the inequalities to equations, which equations they failed to solve because they had not yet mastered the factorization procedure.

The quadratic factorization procedure is one that needs to be done with some understanding. Learners had many conceptual and procedural errors in factorization. Learners showed clearly that their understanding of factorization was in the main incomplete. Therefore far from exploring errors in solving quadratic inequalities per se, the researchers found themselves lodged in exploring errors in the "pathway processes" to solving inequalities such as factorization and dealing with solution to quadratic equations. In as much as learners could not do these we had little hope to study the errors and misconceptions on the target task; solution of quadratic inequalities.

On strategies for factorizing quadratic expression, Yahya and Shahrill, (2015) found out that majority of the participants used trial and error to factorize quadratic expressions. Only two learners used the splitting method. For the second part of the written test, there were four fundamental errors that participants made that instigated them to make errors in answering the given problems. And quite a number of the participants were unable to define the term factorization when asked for its definition during the interview. A lot of them could not relate factorization with distributive law, which is, putting the common terms or linear expression in brackets. A number of participants were inept when asked to state the general formula for quadratic expressions.

Students made errors on multiplication of factors, namely, the wrong use of the third term multiplication factors when doing the splitting method, and the incorrect terms used in finding the multiplication factors used to solve the quadratic expression. Factorizing $2x – 2$ did not yield $2(x +1)$ as was written by a learner. Hence the sum of $3x$ and $2x$ could not be used to replace $5x$ in this case. Furthermore, $3x$ and $2x$ were not the correct multiplication factors for $– 6x$. The correct factor multiplication of $– 6x^2$ to be used here should be $6x$ and $– x$. Didiş, Baş, and Erbaş, (2011), observed that students employed different approaches to factorization depending on the kind or structure of the quadratic equation to be solved, and thus experienced difficulties in different stages of the process.

2.1.1 Incorrect Factors

Certain errors made by the learners revealed they had factorized the quadratic equation into two linear factors incorrectly, and determined the roots incorrectly because they had made false guesses while using the cross-multiplication method. This kind of error occurred mostly, where many students used cross-multiplication as a factoring technique to find the roots of the quadratic equation. The most common error emerged when students used a cross-multiplication method, based on a kind of "guess-and-check" approach, while factorizing the quadratic equations. For example, learners guessed the factors of the constant term incorrectly. Similarly, although learners guessed the factors of the coefficient of x^2 correctly, they were not able to determine the factors of the constant correctly.

In recent years Li, (2011) focused his work on a particular content theme in algebra – basic routines and procedures, which include the algebraic rules, algorithms, and formulas that can be applied to given inputs and yield desired outcomes through finite steps (e.g., the distributive property of multiplication over addition and its extensions, such as the so-called FOIL formula $(a + b)(c + d) = ac + ad + bc + bd$ and identities $(a + b)^2 = a^2 + 2ab + b^2$ and $(a + b)(a – b) = a^2 – b^2$; various established methods or formulas for solving linear and quadratic equations, such as the balancing and backtracking methods, factoring, completing the square, and the quadratic formula).

Makonye and Shingirayi, (2014), on their research on obstacles faced by the learners in the learning of quadratic inequalities used non-probability purposive sampling. The study was conducted at Lamula Jubilee Secondary School in Soweto, Johannesburg, South Africa where one of the researchers taught mathematics. Participants were drawn from a

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Grade 11 mathematics class consisting of 27 learners of whom were 12 boys and 15 were girls. The average age of the learners was 17.2 years. Learners struggled to obtain the correct cognitive structure to solve quadratic inequalities. Learners were at a quandary as they were dealing with many interrelated concepts that they were supposed to sort out; correct factorization, determining the critical values, determining and writing the values of x in inequality form. Sometimes they were required to write the given inequalities in standard form first in which transposing and collecting like terms was necessary. Some learners ignored the inequality signs and handled these as if they were equations. Some learners changed the signs; for example from greater than to less than without any specific logical reason. It was thus quite clear that learners were very anxious and could not handle the problems with inequality signs in a logical manner. Learners clearly lost control of their reasoning and all their work seemed to be guesswork.

Yahya and Shahrill, (2015) while investigating the strategies used by secondary school learners in solving algebraic problems in one of the secondary schools in Brunei Darussalam, conducted in one of the secondary schools in the Belait District (one of the four districts in Brunei Darussalam). The target sample for this study was a class of 21 learners, repeaters who participated in the initial stages of the study. The main reason why the repeaters were chosen to be the sample was because the first author herself taught half of the repeaters when they were in Year 9 and Year 10. Therefore, she knew their weaknesses in the selected areas of algebraic topics. From the 21 learners, only ten repeaters were randomly selected for further participation further in this study. All the learners were similar in socioeconomic status, with the majority of them coming from middle-income families. In the study learners' difficulties in solving questions of changing the subject of a given formula was the most frequent errors made by the learners which included manipulating operations, factorizing linear expressions and in the use of cancellation.

2.1.2 Zero-Product Property

Once learners have factored an expression and work to solve it using the zero-product property (if the product of two numbers is zero, one of the numbers must be zero), they run into additional obstacles. When working to solve an equation such as $x(x+2)=0$, learners sometimes "cancel" the x from both sides (divide by x) leaving $x+2=0$ and $x=0$. They do not see that by doing so, they lose track of the root (Didis & Erbas, 2015; Kotsopoulos, 2007).

The data revealed that some of the learners did not correctly judge whether the quadratic equation to be solved was factorable over some domain, such as rational numbers. For example, some learners attempted to factor the equations; "$x^2 + 2x - 1 = 0$" and "$x^2 + x - 1 = 0$," although they are not factorable, over the rational numbers. For the quadratic equation $x^2 + 2x - 1 = 0$, some learners tried to factorize it as $(x^2 - 1)^2$, $(x + 1)^2$ or $(x - 1)(x + 1)$. For this quadratic equation, all students who had successfully found the roots used the quadratic formula.

2.1.3 Incorrect Factorization in Non- Standard Form

Some learners could not correctly apply the algebraic identity $a^2 - b^2 = (a - b)(a + b)$ to factorize quadratics. Some learners initially moved the term $6x$ to the left side of the equation. They then identified the greatest common factor of the polynomial; $3x$, and rewrote the polynomial using the factored terms. However, although learners put the common term in front of the parentheses correctly, they put the resulting expression inside the parentheses incorrectly. Therefore, when they equaled the factors to zero, they ended up obtaining one of the roots of the quadratic equation incorrectly.

On learners' reasoning in quadratic equations with one unknown (Didiş et al., 2011) sampled 113 learners in four 10th grade classes, and this study was performed in a high school in Antalya, Turkey during the spring term 2009-2010. The study result revealed that factoring the quadratic equations was challenging when they were presented to students in non-standard forms and structures. After looking at the examples of learners' solutions, it can be said that the learners knew some rules (or procedures) related to solving quadratics. However, they tried to apply these rules thinking about neither why they did so, nor whether if what they were doing was mathematically correct. These results give some clues about learners' instrumental understanding of solving quadratic equations with one unknown. Although most of the learners were aware of the correctness of the result, they did not explain the underlying null factor law used to solve the quadratics by factorization. The responses also revealed their misunderstanding of the unknown concept in a quadratic equation and was concluded that the learners' understanding in solving quadratic equations is instrumental (or procedural), rather than relational (or conceptual).

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3. RESEARCH FINDINGS

3.1 Threshold Concepts in Factorization Method

Table 3.1 below indicates the threshold concepts in the performance of symbolic quadratic equations using factorization method in question 1.

Table 3.1: Threshold Concepts in Factorization Q1

Threshold Concepts	Frequency (%)		
	a	**b**	**c**
Factorizing Unstandardized Equation	13 (3.4)	`19 (4.9)	23(6.0)
Equation Equated to a Constant	38 (9.9)	9 (2.3)	2 (0.5)
Zero Product Property	11 (2.9)	9 (2.3)	15 (3.9)
Incorrect Factors	6 (61.6)	24 (6.3)	15 (3.9)
Imposing Linear Structure	43 (11.2)	39 (10.2)	42 (10.9)
NA	161 (41.9)	174 (45.3)	116 (30.2)
Correct solution	112 (29.2)	110 (28.6)	171 (44.5)
Total	**384 (100)**	**384(100)**	**384(100)**

The study established why and what makes learners choose a certain method in solving problems in quadratic equations and functions with one known. Consequently, examining learners' answers in choosing the method of solving the problems provided one of the ways to assess learners' threshold concepts in other methods. This was crucial in the view of the fact that provides the reasons can be identified, and then it should be trouble-free to improve the learners' performance to solve a kin quadratic problem in future. In this regard, when learners solved a quadratic equations, some factorized the equations which were not in standard form in the vein of Q1a; $2q^2 - 8 = 6q$ and Q1c; $c^2 - 14 = 5c$ where 3.4% and 6.0% of the learners lacked comprehension on threshold concepts, as indicated in table above. Learner 19 guessed the sum and product of the equation to be – 3 and – 2 respectively and maintained the value on the right hand side of the equal sign. Learner 287 as shown in the figure below got rid of the square wrongly by squaring the coefficient of x^2 and got 9x. The learner collected like terms to get a wrong value 8. The two learners' lack of comprehension might have affected performance in quadratic equations and functions.

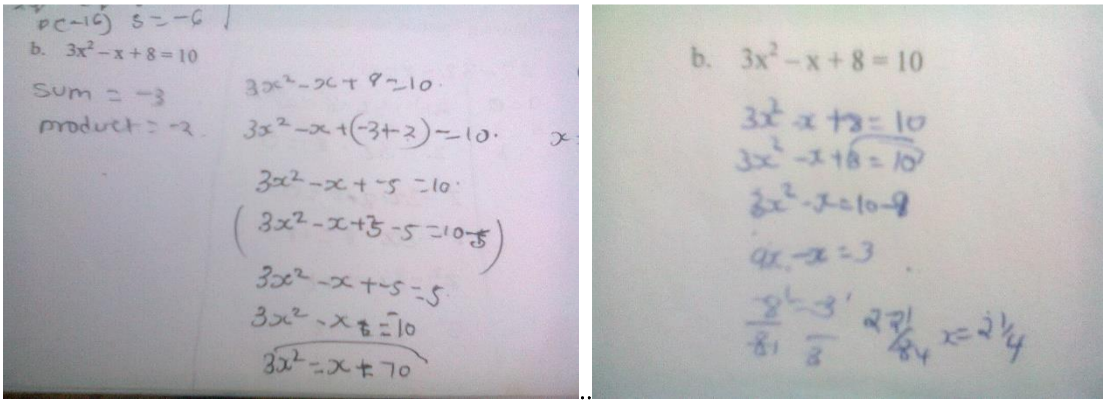

Figure 3.1: Factorizing Unstandardized Equation by Learners 19 and 287

This is a clear indication of lack of fundamental concepts of quadratic equations and functions with one known as designated by 41.9 % 45.3% and 27.3% of the learners in Q1(a, b, c) who considered the method NA. Similarly, Resnick (1982 as cited in Yahya & Shahrill, (2015) stated that threshold concepts in learning are often a result of failure to comprehend the concepts on which procedures are based. Thus, it is important for teachers to develop insights into learner solving in order to identify their threshold concepts.

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Other learners did not rewrite their equations in the form of $ax^2 - bx + c = 0$ and therefore, were unable to factorize equation which is equated to a constant as did by 9.9%, 2.3% and 0.5% of the learners. This implied that lack of comprehension on factorization might have affected performance in quadratic equations and functions. The abstract nature of algebraic expressions posed many problems to learners such as understanding or manipulating them according to accepted rules, procedures, or algorithms. Inadequate comprehension of the uses of the equal sign and its properties when it is used in an equation was a major problem that hindered learners from solving equations correctly, Mamba (2012).

The zero-product property is the main procedure in solving quadratic equations by factoring. As shown in table above, 2.9%, 2.3% and 3.9% of the learners who applied this method had this threshold concept in solving Q1.

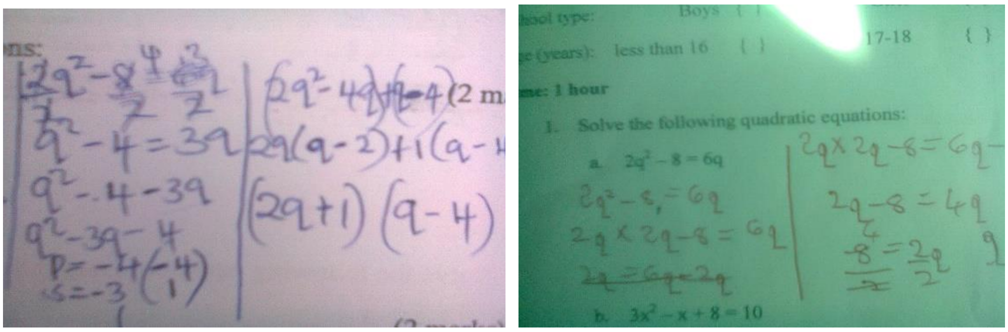

Figure 3.2: Threshold Concept of Zero-Product Property of Learners 74 and 268

The zero-product threshold concept pertains the reasoning that if two factors have a product of zero, then one or both factors must be zero. Therefore, learners would factor one side of an equation to have an equivalent equation such as Q1c; $(c + 2)(c - 7) = 0$. This results in the two equations $(c + 2) = 0 \; and \; (c - 7) = 0$, which can each be solved for x. in figure above, learners 74 and 268 did not have that concept. Though the learner could have first rearranged the equation into standard form, there was nowhere which showed that the learner was aware of zero-product property concept. The literature credits a lack of comprehension on the concept of the zero product property as the reason for why learners solve equations such as $x(x + 2) = 0$ by dividing both sides by *x*, thus losing track of the solution *x* +0, (Kotsopoulos, 2007). Nielsen, (2015), found that learners who were able to factor one side of a quadratic equation were usually able to apply the zero-product property and solve. However, most learners' explanation lacked in completeness, and many provided explanations that appealed to authority regardless of how well they were able to apply the zero-product property.

Even at the post-teaching stage of factorizing a quadratic equation and function with one known, learners still faced lack comprehension of threshold concepts determining the factors. Learners made the wrong use of the third term multiplication factors when doing the splitting method, popularly remembered as product, sum and factors and this might have affected performance in quadratic equations and functions. Matz (1982 cited in Mamba, 2012) explained the persistence of this error that there are two levels of procedures guiding cognitive functioning, namely surface-level procedures, which are the familiar rules of arithmetic and algebra, and deep-level procedures, which create, modify, control and typically guide the surface-level procedures. So, in order to learn quadratic equations and functions a learner should have such a deep-level procedure to overgeneralize numbers; that is, the student must believe that certain procedures work irrespective of the numbers used.

Incorrect factors used in finding the multiplication factors were used to solve the quadratic equations one as indicated by 1.6% of the learners lacked comprehension on the concept in Q1a, 6.3% in Q1b and 3.9% in Q1c and affected performance. Similarly, Lima (2008) and Tall et al. (2014) documented that learners perceive quadratic equations as mere calculations, without paying attention to the unknown as a fundamental characteristic of an equation. Learners mostly focus on the symbolic world to perform operations with symbols. For example, learners used procedural embodiment associated with the exponent of the unknown, and solved the equation by transforming it into $m = 9$ to solve $m^2 = 9$. In this case, learners' use of the procedural embodiments "switching power to roots" resulted in failing to recognize the other root (i.e., $m = -3$). Moreover, they reported that learners attempted to transform quadratic equations into linear equations.

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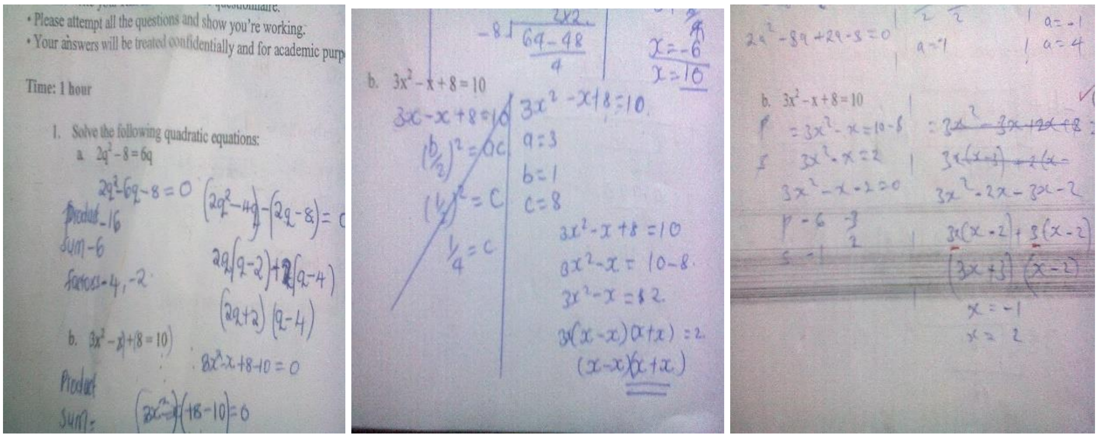

Figure 3.3: Incorrect Factorization by Learners 4, 14 and 45

In question 1a, the factors were 2 and -8, but not only learner 4 had incorrect factors – 4 and – 2 (don't give a product of -16) but also incorrect distribution of a common factor 2q and 2. This implied that the learner lacked comprehension on threshold concepts of the distribution law of factorization; factorized only the first term inside the bracket and forget the second term. Learner 14 performed additive inverse correctly in question 1b, but equated $3x^2 - x = 2$ and proceeded to perform incorrect factorization. Also learner 45 factorized the question in Figure above correctly to obtain -3 and 2 as factors. Hence the learner replaced -x with -2x and -3x which was wrong. Furthermore, -3*x* and -2*x* were not the correct multiplication factors for $-6x^2$. The correct factor multiplication of $-6x^2$ to be used here should be 6*x* and – *x*. However, the learner did not fully comprehend the concept of factorizing because after having factorized the incorrect equation, the term before the parenthesis, 3x and 3 are not common to the two terms inside the bracket and so the final solutions were still incorrect. This implied that the learners didn't comprehend the concept well and affected performance in factorizing quadratic equations and functions. These findings suggest that learners have difficulty with basic multiplication table fact retrieval (Kotsopoulos, 2007).

Factorization threshold concepts increase when the parameter *a* does not equal one (for example in expressions such as $6x^2 - 3x + 2$ and become even more challenging when *a* and/or *c* have multiple factors, leading to many possible factor pairs in expressions such as $20x^2 - 63x + 36$. It is worth noting that the research literature on factoring quadratics attends to factoring when $a, b \text{ and } c$ integers are resulting in expressions that can be factored into binomials with integer coefficients.

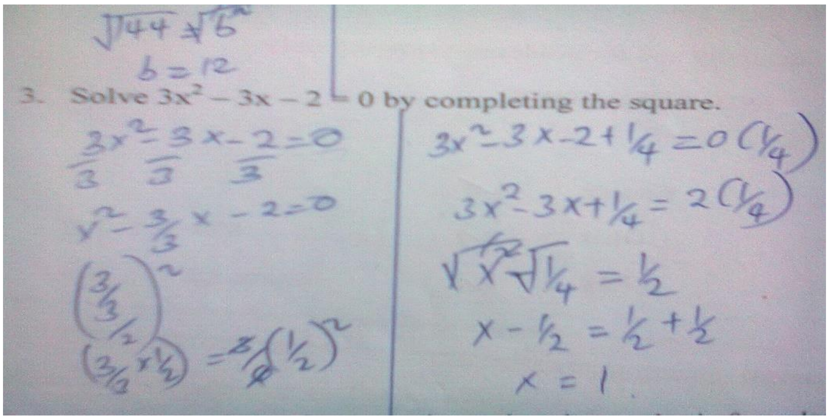

Figure 3.4: Inability to Form Perfect Squares by Learners 138

Learner 138 lacked the concept of dividing through by a common factor. This showed that lack comprehension on the concept of the distributive law which, from a mathematical standpoint, is fundamental not only to the process of factorization in algebra, but also to the reverse process of 'expanding brackets' (Lim, 2000 as cited in Yahya & Shahrill,

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2015). That is, learners have a choice of either a rote-learned cross-multiplication method or a rote learned grouping method when factorizing a quadratic equation; however, neither was ever related to the distribution law. The selection of the method really depended on what their teachers preferred their learners to use.

Because the symbols for the parameters of linear and quadratic equations are often the same, 11.2%, 2.3% and 10.9% of the learners imposed linear structure in Q1 (a, b, and c) respectively.

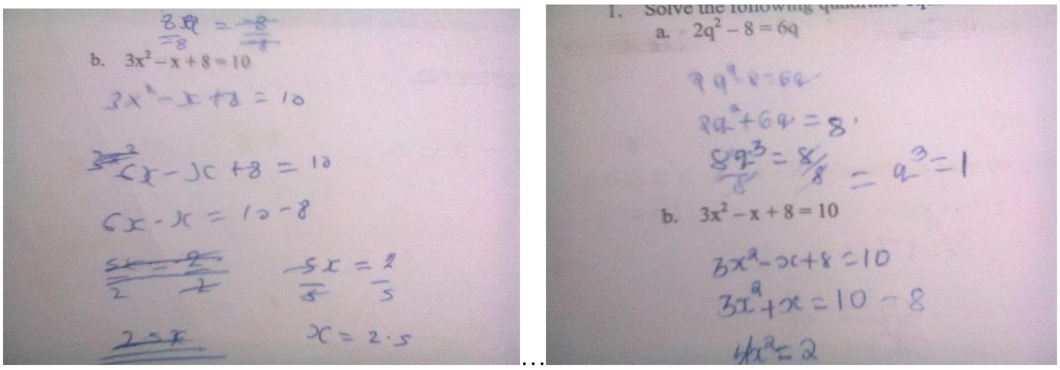

Figure 3.5: Imposing Linear Structure Excerpts from Learners 14 and 18

Previously learnt concepts of linear equations seemed to affect learners 14 and 18 as shown in the excerpts above. Learner 14 interpreted $3x^2$ to be equals to 6x by multiplying 3 which is the coefficient of x^2 and 2 the power of x^2. Student 18 added $2q^2$ and $6q$ to get $8q^3$ forgetting that they were unlike terms in the equations and therefore, lost the properties of quadratic equation.

The figure below shows an excerpt of learner 141 who in the process of dividing the equation by half, wrongfully cancelled $3x^2$ to get 3x hence, linearizing the equation. Learner 187 interpreted x^2 in $3x^2$ to mean $2x$, then wrongly subtracted x to incorrectly remained with $3x$, linearizing it. The results support what Tall et al. (2014) that, while attempting to solve $m^2 = 9$, some students applied the exponent associated with the unknown as if it were the coefficient; that is, m^2 equals to 2m, and learners showed a tendency to use the quadratic formula as the only valid method in solving every quadratic equation. Stacey and MacGregor (1997, as cited in Mamba, 2012) stated that learners may draw on prior learning from other fields to their work with algebraic symbols, e.g., in chemistry, adding oxygen to carbon produces CO_2. Due to similar meanings of 'and' and 'plus' in ordinary language, it is not uncommon for learners to regard 'ab' to mean the same as '$a + b$' because the symbol 'ab' is read as 'a and b' and may be interpreted as '$a + b$', so $3 + x$ may be taken as $3x$. Alternate thinking is that leaners often disregard a difficult question and reformulate it to another easier question such as changing $3 - x$ to 3 (Tall & Thomas 1991 as cited in Mamba, 2012).

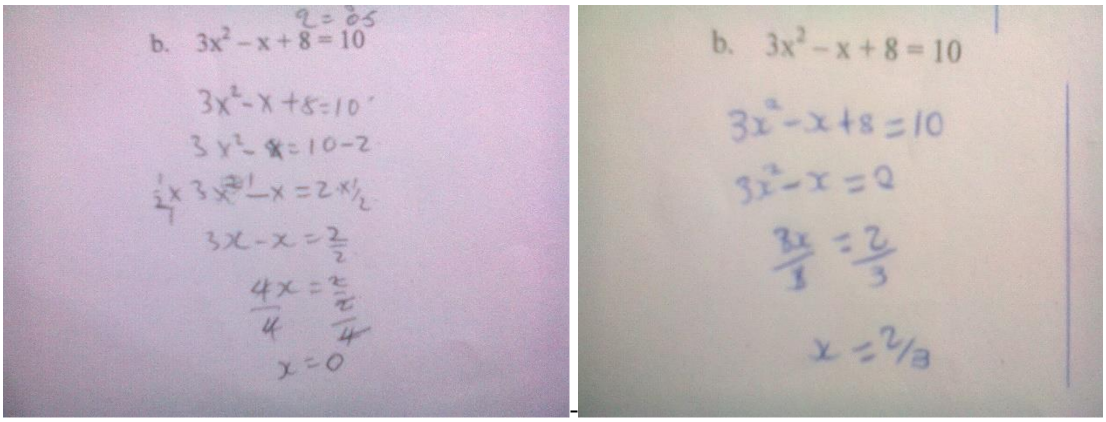

Figure 3.6: Imposed Linear Structure by Learners 141 and 187

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This question 1(b) $3x^2 - x + 8 = 10$; was supposed to be solved as $3x^2 - 3x + 2x - 2 = 0$ to get $3x(x-1)$ + $2(x-1)$ = 0, therefore, $(x-1)(3x+2) = 0$ hence either $(x-1)$ *or* $(3x-2)$ and the answer was $x = 1$ *or* $x = -⅔$. This implied that learners could not comprehend the concepts of solving quadratic equations and functions well and affected performance. Learners either erroneously apply concepts of linear equations to quadratics or use them to "linearize" quadratic equations, Lima and Tall (2010). Didis (2010) interprets this as learners knowing the zero-product property but not being able to apply it appropriately when the structure of the equation is changed. This could also be an example of learners imposing linear structure on a quadratic as they work to solve the equation using techniques that have worked to solve linear equations.

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